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Braid Group and Anyons on a Topologically Nontrivial Surface

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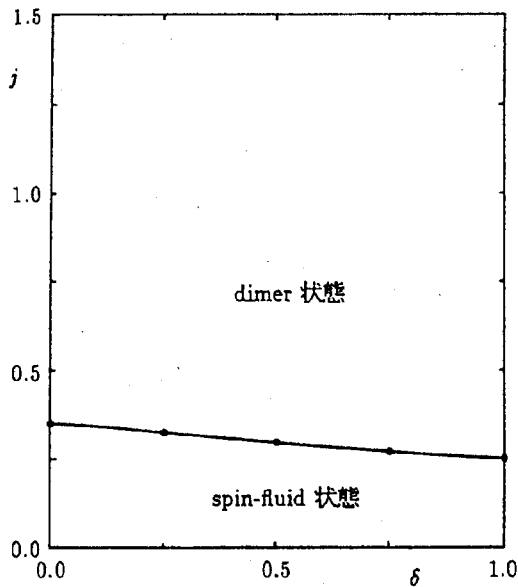


図1. $\gamma=1, 0 \leq \delta < 1$ での基底状態の相図

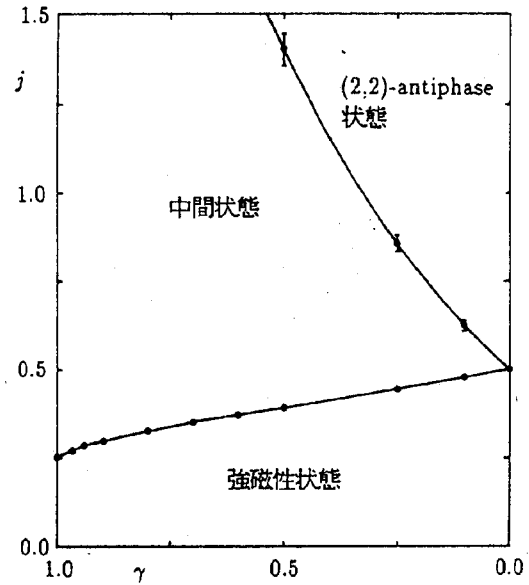


図2. $0 \leq \gamma < 1, \delta=1$ での基底状態の相図

Braid Group and Anyons on a Topologically Nontrivial Surface

Univ. of Utah Yong-Shi Wu

Recently there are interests in understanding the physics of anyons obeying exotic fractional statistics when the 2-d system is put on a topologically nontrivial surface. In this talk I first briefly review the path integral formalism for anyons and fractional statistics in terms of braid group and then report the collaborative work in progress of mine with Dr. Hatzugai and Prof. Kohmoto in this subject. By now it becomes well-known that anyons on a surface S are described by representations of the braid group $B_N(S)$, where N is the total number of anyons. For a planar system, the braid group are generated by the so-called "elementary exchange" operators $\sigma_i (i=1, 2, \dots, N-1)$ satisfying certain defining relations. It has been known that all possible statistics for anyons described by usual (one-component) wave functions are parameterized by an angle $\theta (0 < \theta < 2\pi)$. However, for anyon on a sphere there is one extra relation among σ_i 's, which leads to the restriction $\exp \{i2(N-1)\theta\} = 1$. So θ/π is restricted to be fractional; when N is fixed, allowed values of θ becomes N -dependent and conversely, if θ is fixed, the total anyon number N cannot be arbitrary.

On a cylinder or annulus, the braid group becomes more complicated because even for a single anyon there is a non-contractible loop. So in addition to σ_i 's there are more generators $\rho_i (i=1, 2, \dots, N)$ for the braid group, which moves the i -th particle

along a non-contractible loop. Therefore there are more relations among σ_i and ρ_i and the 1-d representations are described by two angular parameters θ and Φ , where Φ is the magnetic flux threading the cylinder or annulus. By paying attention to satisfying all braid relations for σ_i and ρ_i , we have obtained the correct rules for putting anyons on a lattice with periodic boundary conditions. These rules yield corrections for the existing computer calculations in the literature and clarify some not-well-understood points in them. In particular we have pinned down the extra symmetry condition for a cylinder in contrast to an annulus.

For anyons on a torus, there are even more generators for anyon moving around two independent non-contractible loops. The new extra braid relations require that anyons can be only bosons or fermions if they are described by usual one-component wave functions. So recently Ennassir has suggested to use more-than-one components for the many anyon wave function. In this talk we point out that the necessity of introducing multi-components can be physically understood from the ground state degeneracy on a torus for a rigid 2-d system, such as a fractional quantum Hall system or a chiral spin state, which always supports anyonic excitations. In this way we have seen that anyons have continually given us surprises and excitements.

Strong Correlations and Gauge Fields

東大・物性研 福山 秀敏

電子間強相関の効果は、電子数に対する局所的な制限であるが、これは、量子力学の一般則から電子数と正準共役の関係にある波動関数の位相のゆらぎとして表現される。この位相のゆらぎが、場の理論に於るゲージ場と同様な役割を果たす。一般に、ゆらぎは考えている状態の性質を反映するため、ゲージ場の伝播関数も種々の特徴的な形を持つ。とくに、時間反転対称性を破るカイラル・スピン状態のまわりのゲージ場には Chern-Simons 項と呼ばれる特異項があり、これが分数統計粒子の起源になり得ることが Wen et al.¹⁾ によって、ハイゼンベルグ・スピン模型（ドープされていない状態）に対して議論された。当講演では、これをドープされた状況に拡張することを試みた。即ち、拡張された t-J 模型について分子場近似による相図を決定し、カイラル・スピン状態の出現条件を評価した。更に、このような状態でのゲージ場の Chern-Simons 項の係数、即ち、分数統計の大きさ、がドープの割合に依存することを示した。

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